

Integrating WeBWorK into Textbooks

Sample Exercises

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Abstract

This is a sample article to demonstrate integrating WeBWorK homework problems into content authored with MathBook XML. While technically an example of MathBook's article format, it is intended to closely resemble a chapter of a MathBook book that is divided into sections.

This work was funded with grants from [OpenOregon](#) and the [University of Puget Sound](#).

Rob and Alex worked on the MathBook XML enhancements that make this possible. Mike Gage (University of Rochester) and Geoff Goehle (Western Carolina University) made this possible by enhancing the WeBWorK end, and generally maintaining WeBWorK software.

This article assumes a mild familiarity with both [MathBook XML](#) and [WeBWorK](#). For more information about either project, follow the links.

1 Arithmetic

Some questions with quantitative answers.

Exercise 1.1 (Adding Single-Digit Integers). A simple, but functional example to begin with. If you are just learning how to add, you can test yourself [here](#).

Compute the sum of 9 and 8:

$$9 + 8 = \square$$

Solution. $9 + 8 = 17$.

That was a simple problem. Let's move on.

If you are familiar with WeBWorK, then it may be a surprise to you to be interacting with a WeBWorK problem this way, without having logged in to WeBWorK.

Exercise 1.2 (Declaring a Problem Seed). You can also declare a *seed* to specify a version of any problem that has randomization. Here is the same problem, but with a seed specified.

Compute the sum of 5 and 8:

$$5 + 8 = \square$$

Solution. $5 + 8 = 13$.

Exercise 1.3 (Controlling Randomness). You can code your problem with randomization, but then use a specific seed and WeBWorK's `$envir{problemSeed}` to override that randomization for the purposes of the version that will appear in HTML and print output.

Compute the sum of 1 and 2:

$$1 + 2 = \square$$

Solution. $1 + 2 = 3$.

Exercise 1.4 (Special Answer Checking). One of the strengths of WeBWorK is its ability to give intelligent feedback for incorrect answers.

- There is general feedback for when the student's answer is in an entirely different ballpark from the correct answer. Try entering something like "y".
- There is general feedback for when the student's answer is not in the right form. Try entering " $x^2 * x^3$ ", which, right or wrong, is unsimplified.
- And problems can be written to detect and respond to common mistakes. Try entering an answer where you multiply the two exponents (instead of adding them, which would be correct.)

Use the properties of exponents to simplify $x^5 \cdot x^8 \cdot x^5 \cdot x^8 = \square$

Solution. We *add* the exponents as follows:

$$\begin{aligned} x^5 \cdot x^8 &= x^{5+8} \\ &= x^{13} \end{aligned}$$

Exercise 1.5 (Using Hints). Hints can be inserted into exercises. Whether a hint is visible in the HTML depends on the value of `$showHint` in `PGcourse.pl` in the WeBWorK course that is hosting these exercises. How the hint is displayed in the pdf output is controlled in the usual way that an MBX hint is controlled.

Simplify the expression $\sqrt{45}$.

$$\sqrt{45} = \square$$

Hint. Factor the number inside the radical.

Solution.

$$\sqrt{45} = \sqrt{3^2 \cdot 5} = 3\sqrt{5}$$

or

$$x = \frac{-1 - \sqrt{1^2 - 4(2)(-6)}}{2(2)} = -2$$

3 Open Problem Library

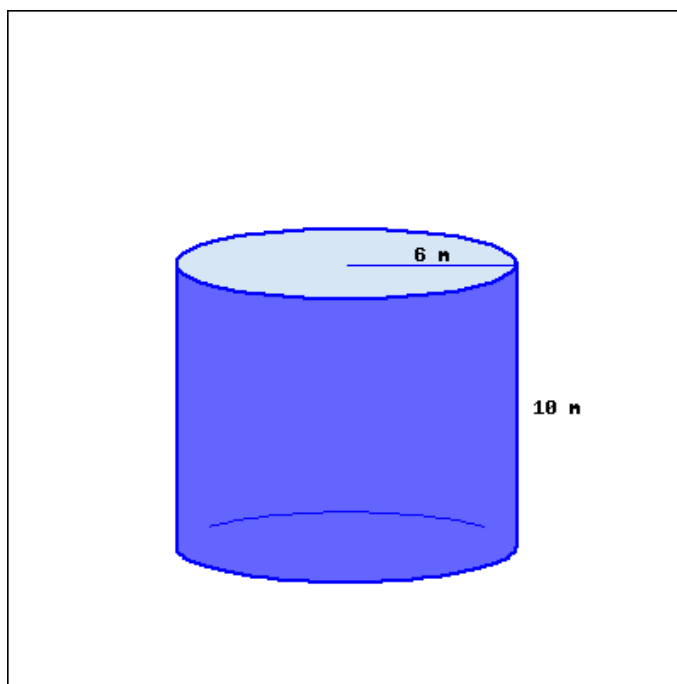
The [Open Problem Library](#) (OPL) is a repository of curated WeBWorK problems. At last count it had something like 30,000 problems, searchable by topic.

Most of the problems in this sample chapter have their source authored within the same document as the narrative. However the problems in this section are from the OPL and elsewhere on the server that is hosting the WeBWorK calculations. For HTML output we can connect to the server and render the problem here in an interactive fashion. For print/PDF we use the `mbx` script to connect to the server to produce the LaTeX code for the problem. We can use the `exercise.text.*` switches to control which parts of the problem (statements, solutions, hints) are incorporated.

Problems do not have to be in the OPL to reside on a server; any problem that is accessible from the WeBWorK course which is hosting the WeBWorK computation may be used this way.

Exercise 3.1 (Cylinder Volume). This problem is one that Portland Community College has donated to the Open Problem Library.

A cylinder's base's radius is 6 m, and its height is 10 m.



- This cylinder's volume, in terms of π , is _____. (Type **pi** for π .)
- This cylinder's volume, rounded to the hundredth place, is _____.

(Use $\mathbf{m^3}$ for cubic meters.)

Solution:

We use r to represent the base's radius, and h to represent the cylinder's height.

A cylinder's volume formula is $V = (\text{base area}) \cdot \text{height}$. A cylinder's base is a circle, with its area formula $A = \pi r^2$.

Putting together these two formulas, we have a cylinder's volume formula:

$$V = \pi r^2 h$$

Throughout these computations, all quantities have units attached, and we only show them in the final step.

a. Using the volume formula, we have:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \cdot 6^2 \cdot 10 \\ &= \pi \cdot 360 \\ &= 360\pi \text{ m}^3 \end{aligned}$$

Don't forget the volume unit m^3 .

b. To find the decimal version, we replace π with its decimal value, and we have:

$$\begin{aligned} V &= 360\pi \\ &\approx 360 \cdot 3.14 \dots \\ &\approx 1130.97 \text{ m}^3 \end{aligned}$$

Don't forget the volume unit m^3 .

[BasicAlgebra/Geometry/CylinderVolume10.pg](#)

Seed: 8435

Exercise 3.2 (Grandpa's Cat: a Cautionary Tale). Not all problems that come from the OPL were written with MBX in mind, and they don't all fit into MBX space. This example illustrates that this should be taken into consideration when using OPL problems. Some of the graphs here are not fully visible in the web version of the article.

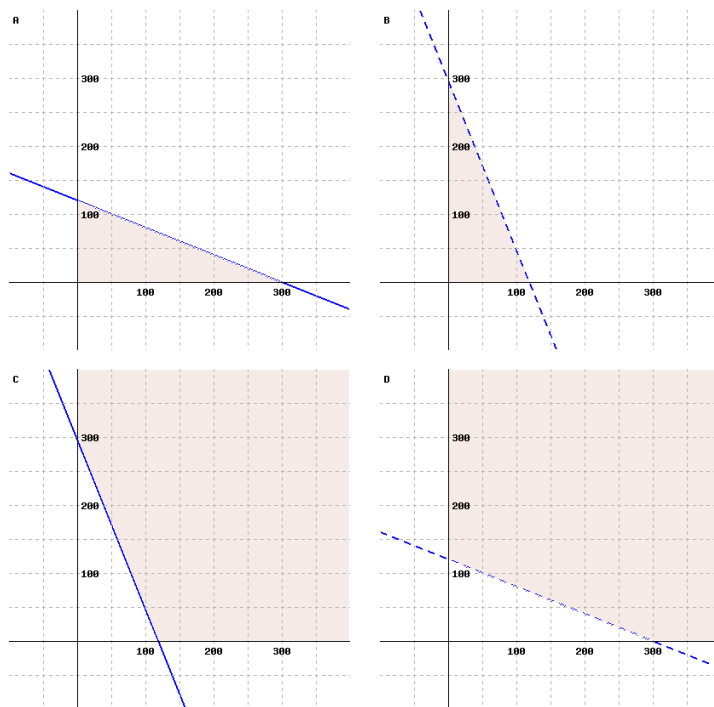
You fed your grandpa's cat while he was on vacation. When he was back, he took out a huge back of coins, including quarters and dimes. He said you can take as many coins as you want, but the total value must be lower than \$30.00.

Write an inequality to model this situation, with x representing the number of quarters you will take, and y representing the number of dimes.

Which of the following graphs represent this situation?

The correct graph is:

- A
- B
- C
- D



Solution: Since each quarter is worth \$0.25, x quarters are worth $0.25x$ dollars.

Similarly, since each dime is worth \$0.10, y dimes are worth $0.1y$ dollars.

Together, x quarter and y dime are worth a total of $0.25x + 0.1y$ dollars.

The total value of coins you take must be less than \$30.00, implying $0.25x + 0.1y < 30$. Note that the symbol is $<$, because the total value cannot be \$30.00.

To graph the inequality $0.25x + 0.1y < 30$, we first graph the line $0.25x + 0.1y = 30$. Because decimals are involved, it's easier to graph this line by finding its x and y intercepts.

To find its x -intercept, we substitute $y = 0$ into $0.25x + 0.1y = 30$, and

we have:

$$\begin{aligned}0.25x + 0.1y &= 30 \\0.25x + 0.1 \cdot 0 &= 30 \\0.25x &= 30 \\ \frac{0.25x}{0.25} &= \frac{30}{0.25} \\ x &= 120\end{aligned}$$

This line's x -intercept is $(120, 0)$.

Similarly, we can find the y -intercept is $(0, 300)$. This is left as an exercise.

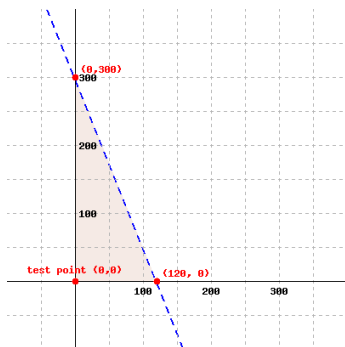
Now we can graph this line. Note that the line is dashed, because the inequality symbol is $<$.

Next, we need to decide which side to shade. We use $(0, 0)$ as the test point. Substituting $x = 0$ and $y = 0$ into the inequality, and we have:

$$\begin{aligned}0.25x + 0.1y &< 30 \\0.25 \cdot 0 + 0.1 \cdot 0 &< 30 \\0 &< 30\end{aligned}$$

This is true, so we shade the side with the point $(0, 0)$.

We can see the correct answer is B.



[BasicAlgebra/EquationsAndInequalitiesInTwoVariables/GraphTwoVariableInequality110.pg](#)

Seed: 123567890

4 Antidifferentiation

4.1 A Few More Features

This subsection demonstrates a few more features.

The derivative and antiderivative of a function can be understood through study of their graphical relationships.

Definition 4.1 (Antiderivative of a Function). Suppose that $f(x)$ and $F(x)$ are two functions such that

$$F'(x) = f(x).$$

Then we say F is an **antiderivative** of f .

The Fundamental Theorem of Calculus is one of the high points of a course in single-variable course.

Theorem 4.2 (The Fundamental Theorem of Calculus). *If $f(x)$ is continuous, and the derivative of $F(x)$ is $f(x)$, then*

$$\int_a^b f(x) dx = F(b) - F(a)$$

Proof. Left to the reader. □

We state an equivalent version of the FTC, which is less-suited for computation, but which perhaps is a more interesting theoretical statement.

Corollary 4.3. *Suppose $f(x)$ is a continuous function. Then*

$$\frac{d}{dx} \int_a^x f(t) dt = f(x) \tag{4.1}$$

Proof. We simply take the indicated derivative, applying Theorem [Theorem 4.2](#) at [\(4.2\)](#).

$$\frac{d}{dx} \int_a^x f(t) dt = \frac{d}{dx} (F(x) - F(a)) \tag{4.2}$$

$$\begin{aligned} &= \frac{d}{dx} F(x) - \frac{d}{dx} F(a) \\ &= f(x) - 0 = f(x) \end{aligned} \tag{4.3}$$

□

4.2 WeBWorK Exercises

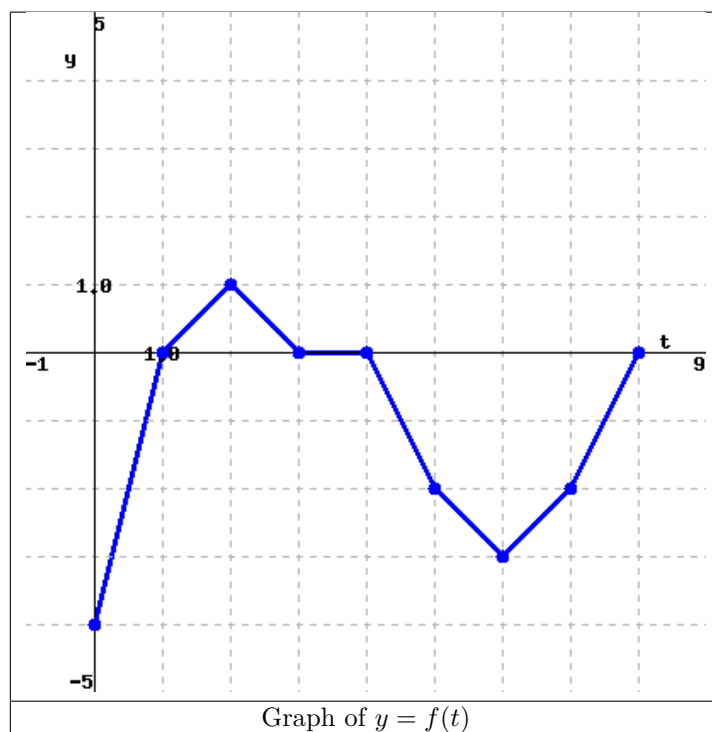
This first problem in this list is coming from the WeBWorK Open problem Library. One implication of this is that we might want to provide some commentary that connects the problem to the text. The other two ask for essay answers, which would be graded by an instructor, so there is no opportunity to provide answer.

1. (Graphical Antiderivatives) Consult [Definition 4.1](#) and the [The Fundamental Theorem of Calculus](#) to assist you with the following problem.

Let $f(t)$ be the piecewise linear function with domain $0 \leq t \leq 8$ shown in the graph below (which is determined by connecting the dots). Define a function $A(x)$ with domain $0 \leq x \leq 8$ by

$$A(x) = \int_0^x f(t) dt.$$

Notice that $A(x)$ is the net area under the function $f(t)$ for $0 \leq t \leq x$. If you click on the graph below, a full-size picture of the graph will open in another window.



(A) Find the following values of the function $A(x)$.

$A(0) = \underline{\hspace{2cm}}$

$A(1) = \underline{\hspace{2cm}}$

$A(2) = \underline{\hspace{2cm}}$

$A(3) = \underline{\hspace{2cm}}$

$A(4) = \underline{\hspace{2cm}}$

$A(5) = \underline{\hspace{2cm}}$

$A(6) = \underline{\hspace{2cm}}$

$A(7) = \underline{\hspace{2cm}}$

$A(8) = \underline{\hspace{2cm}}$

(B) Use **interval notation** to indicate the interval or union of intervals where $A(x)$ is increasing and decreasing.

$A(x)$ is increasing for x in the interval $\underline{\hspace{2cm}}$

$A(x)$ is decreasing for x in the interval $\underline{\hspace{2cm}}$

(C) Find where $A(x)$ has its maximum and minimum values.

$A(x)$ has its maximum value when $x = \underline{\hspace{2cm}}$

$A(x)$ has its minimum value when $x = \underline{\hspace{2cm}}$

Solution:

Solution:

(A) The net area under the function $y = f(t)$ can be calculated by sub-

dividing the region between the t -axis and the function into rectangles, triangles, and trapezoids whose area can be easily calculated.

(B) The function $A(x)$ is increasing when $f(t)$ is above the t axis, decreasing when $f(t)$ is below the t axis, and is neither increasing nor decreasing when $f(t)$ is on the t axis.

(C) To find the maximum and minimum values of the function $A(x)$, all you need to do is to find the largest and smallest values from your answer to part (A).

[Library/Rochester/setIntegrals4FTC/S05.03.FundThmCalc.PTP01.pg](#)

Seed: 123567890

2. (Every Continuous Function has an Antiderivative) WeBWorK problems can allow for open-ended essay responses that are intended to be assessed later by the instructor. For anonymous access, no text field is provided. But if this problem were used within WeBWorK as part of a homework set, users could submit an answer.

Explain how we can use [Corollary 4.3](#) to say that every continuous function *always* has a derivative. (And we will demonstrate here that you can use a macro from docinfo: $\int_1^2 \frac{1}{x} dx = \ln(2)$. It will work in the WeBWorK problem, regardless of whether you are using images, MathJax, or hardcopy.)

[Essay Answer]

3. (Inverse Processes) “Differentiation and integration are inverse processes.” Cite *specific results* from this section in an explanation of how they justify this (somewhat imprecise) claim.

[Essay Answer]

5 PGML Formatting Calisthenics

This section has one exercise, designed to test various PGML formatting rules in an exercise with no purpose. Consult the source to see how the special characters and formatting are realized.

Exercise 5.1 (PGML Formatting). Smart double quotes: “Life is about making an impact, not making an income.”

Smart single quotes: ‘Whatever the mind of man can conceive and believe, it can achieve.’

Emphasis: *very important*

Alert: **do not** do it

Verbatim: code, with special characters: a * , a \, and a #

Some pre-formatted text
with an indented line
and an out-dented line

Some of L^AT_EX’s special characters:

\$ % ^ & _ { } ~ \

Some characters that need special handling for PG conversion to HTML or L^AT_EX:

A non breaking space (invisible where a hyphen should be)

An ndash—right here.

An mdash—right here.