

Integrating WeBWorK into Textbooks

Sample Exercises

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Abstract

This is a sample article to demonstrate integrating WeBWorK homework problems into content authored with PreTeXt. While technically an example of PreTeXt's `article` format, it is intended to closely resemble a chapter of a PreTeXt `book` that is divided into sections.

This work was funded with grants from [OpenOregon](#) and the [University of Puget Sound](#).

Rob Beezer (University of Puget Sound) and Alex Jordan (Portland Community College) worked on the PreTeXt enhancements that make this possible. Mike Gage (University of Rochester), Geoff Goehle (Western Carolina University), and Alex Jordan made this possible by enhancing the WeBWorK end, and generally maintaining WeBWorK software.

This article assumes a mild familiarity with both [PreTeXt](#) and [WeBWorK](#). For more information about either project, follow the links.

1 Arithmetic

Some questions with quantitative answers.

Checkpoint 1.1 (Adding Single-Digit Integers). A simple, but functional example to begin with. If you are just learning how to add, you can test yourself here.

Compute the sum of 6 and 1:

$$6 + 1 = \underline{\quad}$$

Solution. $6 + 1 = 7$.

That was a simple problem. Let's move on.

If you are familiar with WeBWorK, then it may be a surprise to you to be interacting with a WeBWorK problem this way, without having logged in to WeBWorK.

Checkpoint 1.2 (Declaring a Problem Seed). You can also declare a *seed* to specify a version of any problem that has randomization. Here is the same problem, but with a *seed* specified.

Compute the sum of 5 and 8:
 $5 + 8 = \underline{\hspace{2cm}}$

Solution. $5 + 8 = 13$.

Checkpoint 1.3 (Controlling Randomness). You can code your problem with randomization, but then use a specific *seed* and WeBWorK's `$envir{problemSeed}` to override that randomization for the purposes of the version that will appear in HTML and print output.

Compute the sum of 1 and 2:
 $1 + 2 = \underline{\hspace{2cm}}$

Solution. $1 + 2 = 3$.

Checkpoint 1.4 (Special Answer Checking). One of the strengths of WeBWorK is its ability to give intelligent feedback for incorrect answers.

- There is general feedback for when the student's answer is in an entirely different ballpark from the correct answer. Try entering something like "y".
- There is general feedback for when the student's answer is not in the right form. Try entering " $x^2 * x^3$ ", which, right or wrong, is unsimplified.
- And problems can be written to detect and respond to common mistakes. Try entering an answer where you multiply the two exponents (instead of adding them, which would be correct.)

Use the properties of exponents to simplify $x^5 x^3$.
 $x^5 x^3 = \underline{\hspace{3cm}}$

Solution. We *add* the exponents as follows:
 $x^5 x^3 = x^{5+3}$
 $= x^8$

Checkpoint 1.5 (Using Hints). Hints can be inserted into exercises. Whether a hint is visible in the HTML depends on the value of `$showHint` in `PGcourse.pl` in the WeBWorK course that is hosting these exercises. How the hint is displayed in the pdf output is controlled in the usual way that an PTX hint is controlled.

Simplify the expression $\sqrt{48}$.
 $\sqrt{48} = \underline{\hspace{3cm}}$

Hint. Factor the number inside the radical.

Solution. $\sqrt{48} = \sqrt{4^2 \cdot 3} = 4\sqrt{3}$

Checkpoint 1.6 (No Randomization). This problem has no randomization at all, not even if it were exported to be part of a problem set on a WeBWorK server. As such, it really doesn't need any lines of Perl code in its setup, so you have the option of skipping that part of the authoring process.

$1 + 2 = \underline{\hspace{2cm}}$

For more about problems that do not require any randomization, see the [PTX Author Guide](#).

2 The Quadratic Formula

In the previous section, we saw relatively simple WeBWorK questions. This section demonstrates how even very complicated WeBWorK problems can still behave well.

Here is a theorem that gives us a formula for the solutions of a second-degree polynomial equation. Note later how the WeBWorK problem references the theorem by its number. This seemingly minor detail demonstrates the degree to which WeBWorK and PreTeXt have been integrated.

Theorem 2.1 (Quadratic Formula). *Given the second-degree polynomial equation $ax^2 + bx + c = 0$, where $a \neq 0$, solutions are given by*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Proof.

$$\begin{aligned} ax^2 + bx + c &= 0 \\ ax^2 + bx &= -c \\ 4ax^2 + 4bx &= -4c \\ 4ax^2 + 4bx + b^2 &= b^2 - 4ac \\ (2ax + b)^2 &= b^2 - 4ac \\ 2ax + b &= \pm\sqrt{b^2 - 4ac} \\ 2ax &= -b \pm \sqrt{b^2 - 4ac} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \square \end{aligned}$$

Checkpoint 2.2 (Solving Quadratic Equations).

Part 1. Consider the quadratic equation given by

$$4x^2 - 21x - 18 = 0.$$

First, identify the coefficients for the quadratic equation using the standard form from Theorem 2.1.

$$a = \underline{\hspace{2cm}}, \quad b = \underline{\hspace{2cm}}, \quad c = \underline{\hspace{2cm}}$$

Solution. Take the coefficient of x^2 for the value of a , the coefficient of x for b , and the constant for c . *In this case*, they are $a = 4$, $b = -21$, $c = -18$.

Part 2. Using the quadratic formula, solve $4x^2 - 21x - 18 = 0$.

$$x = \underline{\hspace{4cm}} \quad \text{or} \quad x = \underline{\hspace{4cm}}$$

Solution. Recall that the quadratic formula is given in Theorem 2.1.

You already identified $a = 4$, $b = -21$, and $c = -18$, so the results are:

$$\begin{aligned} x &= \frac{-(-21) + \sqrt{(-21)^2 - 4 \cdot 4 \cdot (-18)}}{2(4)} = 6 \\ \text{or} \\ x &= \frac{-(-21) - \sqrt{(-21)^2 - 4 \cdot 4 \cdot (-18)}}{2(4)} = -\frac{3}{4} \end{aligned}$$

3 Open Problem Library

The [Open Problem Library](#) (OPL) is a repository of curated WeBWorK problems. At last count it had something like 30,000 problems, searchable by topic.

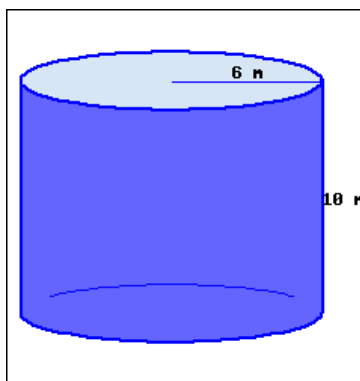
Most of the problems in this sample chapter have their source authored within the same document as the narrative. However the problems in this

section are from the OPL and elsewhere on the server that is hosting the WeBWorK caclulations. For HTML output we can connect to the server and render the problem here in an interactive fashion. For print/PDF we use the `mbx` script to connect to the server to produce PreTeXt code for the problem. We can use the `exercise.text.*` switches to control which parts of the problem (statements, solutions, hints) are incorporated.

Problems do not have to be in the OPL to reside on a server; any problem that is accessbile from the WeBWorK course which is hosting the WeBWorK computation may be used this way.

Checkpoint 3.1 (Cylinder Volume). This problem is one that Portland Community College has donated to the Open Problem Library.

A cylinder's base's radius is 6 m, and its height is 10 m.



- This cylinder's volume, in terms of π , is _____.
- This cylinder's volume, rounded to the hundredth place, is _____.

Solution. We use r to represent the base's radius, and h to represent the cylinder's height.

A cylinder's volume formula is $V = (\text{base area}) \cdot \text{height}$. A cylinder's base is a circle, with its area formula $A = \pi r^2$.

Putting together these two formulas, we have a cylinder's volume formula:

$$V = \pi r^2 h$$

Throughout these computations, all quantities have units attached, and we only show them in the final step.

- Using the volume formula, we have:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \cdot 6^2 \cdot 10 \\ &= \pi \cdot 360 \\ &= 360\pi \text{ m}^3 \end{aligned}$$

Don't forget the volume unit m^3 .

- b. To find the decimal version, we replace π with its decimal value, and we have:

$$\begin{aligned}V &= 360\pi \\ &\approx 360 \cdot 3.14\dots \\ &\approx 1130.97 \text{ m}^3\end{aligned}$$

Don't forget the volume unit m^3 .

Warning 3.2 (Incompatible Problems). Not all problems that come from the OPL are compatible with PreTeXt. The reasons vary. Perhaps the problem uses older WeBWorK macros that cannot be augmented to provide PreTeXt output. Perhaps the problem nests a table within a table, which PreTeXt will not support. Perhaps there are graphics in the problem that are not sized appropriately for PreTeXt output and there's nothing you can do about it. And there are many more reasons.

Unfortunately the variety of reasons for which a problem may be incompatible means that at present, there is no way to validate a problem automatically. Using an incompatible problem might not raise any errors at any stage of the PreTeXt processing, but large chunks of the problem may be missing, especially in print output. So we recommend that you review all output modes when using an OPL problem. Chances are that if it is behaving in print, it will behave elsewhere. But that is an unproven theory at this point.

4 Antidifferentiation

4.1 A Few More Features

This subsection demonstrates a few more features.

Definition 4.1 (Antiderivative of a Function). Suppose that $f(x)$ and $F(x)$ are two functions such that

$$F'(x) = f(x).$$

Then we say F is an **antiderivative** of f .

The Fundamental Theorem of Calculus is one of the high points of a course in single-variable calculus.

Theorem 4.2 (The Fundamental Theorem of Calculus). *If $f(x)$ is continuous, and the derivative of $F(x)$ is $f(x)$, then*

$$\int_a^b f(x) dx = F(b) - F(a)$$

Proof. Left to the reader. □

We state an equivalent version of the FTC, which is less-suited for computation, but which perhaps is a more interesting theoretical statement.

Corollary 4.3. *Suppose $f(x)$ is a continuous function. Then*

$$\frac{d}{dx} \int_a^x f(t) dt = f(x) \tag{4.1}$$

Proof. We simply take the indicated derivative, applying [Theorem 4.2](#) at [\(4.2\)](#).

$$\frac{d}{dx} \int_a^x f(t) dt = \frac{d}{dx} (F(x) - F(a)) \tag{4.2}$$

$$\begin{aligned}
&= \frac{d}{dx} F(x) - \frac{d}{dx} F(a) \\
&= f(x) - 0 = f(x)
\end{aligned}
\tag{4.3}$$

□

WeBWorK Exercises

The first problem in this list is coming from the WeBWorK Open Problem Library. One implication of this is that we might want to provide some commentary that connects the problem to the text. The other two ask for essay answers, which would be graded by an instructor, so in the HTML output there is no opportunity to provide an answer.¹ (Antiderivatives) Consult [Definition 4.1](#) and the [The Fundamental Theorem of Calculus](#) to assist you with the following problem.

$$\int_0^3 (5e^x + 2 \sin x) dx = \underline{\hspace{10cm}}$$

Solution. SOLUTION

$$\begin{aligned}
\int_0^3 (5e^x + 2 \sin x) dx &= \left. 5e^x - 2 \cos x \right|_0^3 \\
&= (5e^3 - 2 \cos 3) - (5e^0 - 2 \cos 0) \\
&= 5e^3 - 2 \cos 3 - 3
\end{aligned}$$

Unless the OPL has changed this problem out from under us, note the “SOLUTION” appearing in the solution. That is hard-coded into the OPL version of the problem. This is an example of something undesirable that may happen when using OPL problems that were not originally written with PreTeXt in mind.

2. (Every Continuous Function has an Antiderivative) WeBWorK problems can allow for open-ended essay responses that are intended to be assessed later by the instructor. For anonymous access, no text field is provided. But if this problem were used within WeBWorK as part of a homework set, users could submit an answer.

Explain how we can use Corollary 4.3 to say that every continuous function *always* has a derivative. (And we will demonstrate here that you can use a macro from `docinfo`: $\int_1^2 \frac{1}{x} dx = \ln(2)$. It will work in the WeBWorK problem, regardless of whether you are using images, MathJax, or hardcopy.)

3. (Inverse Processes) “Differentiation and integration are inverse processes.” Cite *specific results* from this section in an explanation of how they justify this (somewhat imprecise) claim.

5 PGML Formatting Calisthenics

This section has one exercise, designed to test various PGML formatting rules in an exercise with no purpose. Consult the source to see how the special characters and formatting are realized.

Checkpoint 5.1 (PGML Formatting). Smart double quotes: “Life is about making an impact, not making an income.”

Smart single quotes: ‘Whatever the mind of man can conceive and believe, it can achieve.’

Emphasis: *very important*

Alert: **do not** do it

Verbatim: code, with special characters: a * , a \, and a #

Some pre-formatted text
with an indented line
and an out-dented line

Some of L^AT_EX’s special characters:

\$ % ^ & _ { } ~ \

Some characters that need special handling for PG conversion to HTML or L^AT_EX:

A non breaking space (invisible where a hyphen should be)

An ndash—right here.

An mdash—right here.

6 Multiple Choice

While free-response questions are generally preferred, sometimes the nature of a question lends itself to multiple choice.

Checkpoint 6.1 (Drop-down/Popup). Note also that the solution to this problem uses an external link.

The number $\sqrt{2}$ (is is not) rational.

Solution. If $\sqrt{2}$ were rational, then $\sqrt{2} = \frac{p}{q}$, with p and q coprime. But then $2q^2 = p^2$. By [the Fundamental Theorem of Arithmetic](#), the power of 2 dividing the left side is odd, while the power of 2 dividing the right side is even. This is a contradiction, so $\sqrt{2}$ is not rational.

Checkpoint 6.2 (Choose one). Which of the following suggest that differentiation and integration are inverse processes?

- The Quadratic Formula
- The Fundamental Theorem of Calculus
- The Fundamental Theorem of Arithmetic
- None of these

Solution. The correct answer is The Fundamental ... of Calculus.

7 Tables

Although a WeBWorK problem written in PreTeXt source can’t have a “table” (which would be captioned and cause trouble with free-standing PG output), they can have a “tabular” (which must be a child of a “sidebyside”). Tabulars can be made using the existing syntax and options that PTX offers for table-making, with some exceptions. As of summer 2017, the exceptions are that individual cells cannot have bottom border attributes and individual columns

cannot have `top` border attributes. Also, while PTX table borders are generally `minor`, `medium`, or `major`, when these borders are used in a WeBWorK problem that is rendered as a hard copy problem from within WeBWorK, the only thickness option corresponds to `minor`. Several more features (like rules in general) do not carry through to static output (print or HTML-preview). However we hope this is only temporary.

Checkpoint 7.1 (Complete this Table). Complete this multiplication table.

\times	8	8
2		
9		

Solution.

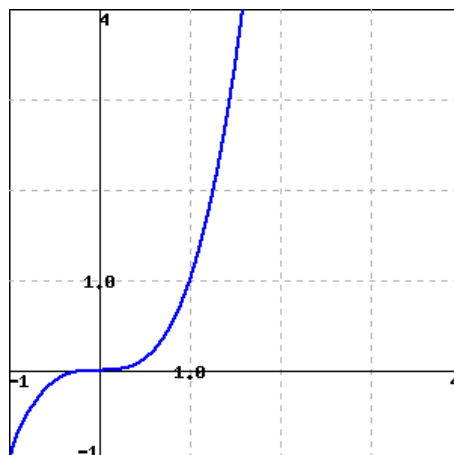
\times	8	8
2	16	16
9	72	72

8 Graphics in Exercises

It is natural to want to write exercises that rely on graphics. For example, an exercise might produce a chart of some kind, and ask the reader to extract some information from that chart.

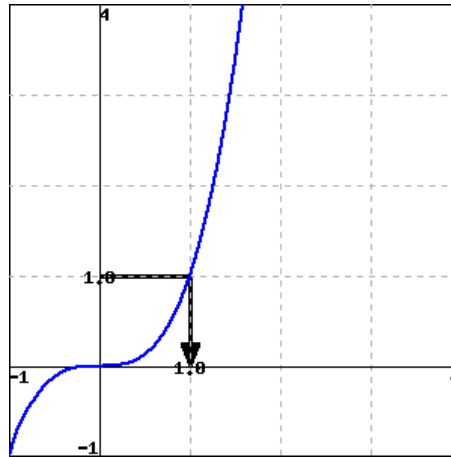
At present, this can be done using the same mechanism one would use to insert a graphic into a WeBWorK problem when writing regular WeBWorK problems. (And this is what is done below.) In the future, there will be a nicer option that will be able to make use of PTX's `latex-image-code`. This requires some WeBWorK development first, which is underway.

Checkpoint 8.1 (Solve using a graph). The graph below is a graph of $y = f(x)$. Use the graph to solve the equation $f(x) = 1$.



$x =$ _____

Solution. The graph reveals that the only solution to $f(x) = 1$ is $x = 1$.



9 Subject Area Templates

This section samples the subject area template problems found on the WeB-WorK wiki at <http://webwork.maa.org/wiki/SubjectAreaTemplates>.

Checkpoint 9.1 (Answer is a number or a function).

- a. Suppose the correct answer is 1.

■ _____

- b. Suppose the correct answer is $\frac{x^7 + 7}{x}$.

■ _____

Solution. Solution explanation goes here.

Checkpoint 9.2 (Answer is a function with domain issues).

- a. Suppose the correct answer is $\sqrt{x - 4}$.

■ _____

- b. Suppose the correct answer is $\ln\left(\left|\frac{x}{x - 4}\right|\right)$.

■ _____

Solution. Solution explanation goes here.

Checkpoint 9.3 (Multiple choice by popup, radio buttons). My favorite color is Red Blue Green .

My favorite color is

- Red
- Blue
- Green
- None of these

Solution. The correct answer is Blue.
 The correct answer is Blue.

Checkpoint 9.4 (Tables). A table with minimal XML source.

A	B	C
D	E	F
G	H	I

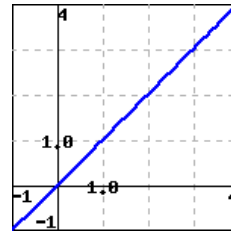
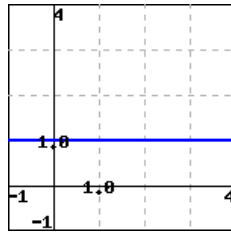
Finish this table.

1	two	$[\pi]$
$I + I + I + I$	_____	six
$2^3 - 1$	VIII	

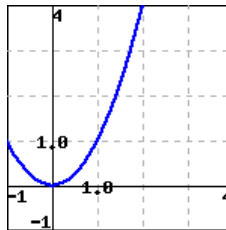
Solution. The missing number is 5.

10 Stress Tests

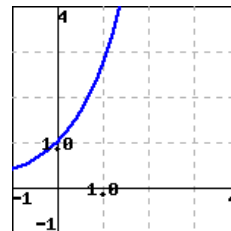
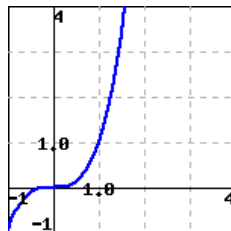
Checkpoint 10.1 (PTX problem source with server-generated images).
 Part 1.

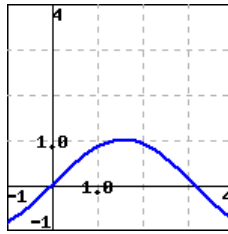


Solution.

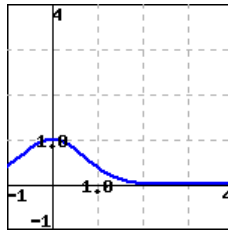


Part 2.





Solution.



Checkpoint 10.2 (Checking Proper Indentation In Lists).

1. Simple item
2. Simple item
3. Simple item

1. Simple item
2. Simple item
3. Simple item

Text before.

1. Simple item
2. Simple item
3. Simple item

And after.

1. Structured item
2. Structured item
3. Structured item

Text before.

1. Structured item
2. Structured item
3. Structured item

And after.

1. Structured item

2.

a. Sublist Item

b. Sublist Item

c. Sublist Item

3. Structured item

Text before.

1. Structured item

2. Text before.

a. Sublist Item

b. Sublist Item

c. Sublist Item

And after.

3. Structured item

And after.

1. Structured item

Second paragraph

2.

a. Sublist Item

Second paragraph

b. Sublist Item

Second paragraph

c. Sublist Item

Second paragraph

3. Structured item

Second paragraph

Text before.

1. Structured item

Second paragraph

2. Text before.

a. Sublist Item

Second paragraph

b. Sublist Item

Second paragraph

- c. Sublist Item
Second paragraph

And after.

- 3. Structured item
Second paragraph

And after.

1.

- 1 + 1 = 2
Structured item
Second
1 + 1 = 2
paragraph

2.

- a. Sublist
1 + 1 = 2
Item
Second paragraph
1 + 1 = 2
- b.
1 + 1 = 2
Sublist Item
Second
1 + 1 = 2
paragraph
- c. Sublist Item
1 + 1 = 2
1 + 1 = 2
Second paragraph

3. Structured

- 1 + 1 = 2
item
Second paragraph
1 + 1 = 2

Text before.

1. Structured

- 1 + 1 = 2
item
Second paragraph
1 + 1 = 2

2.

$$1 + 1 = 2$$

Text before.

a. Sublist

$$1 + 1 = 2$$

Item

Second paragraph

$$1 + 1 = 2$$

b.

$$1 + 1 = 2$$

Sublist Item

Second

$$1 + 1 = 2$$

paragraph

c. Sublist Item

$$1 + 1 = 2$$

$$1 + 1 = 2$$

Second paragraph

And

$$1 + 1 = 2$$

after.

3. Structured item

$$1 + 1 = 2$$

$$1 + 1 = 2$$

Second paragraph

And after.

Text before.

1. Structured

$$1 + 1 = 2$$

$$2 = 2$$

item

Second paragraph

$$1 + 1 = 2$$

$$2 = 2$$

2.

$$1 + 1 = 2$$

$$2 = 2$$

Text before.

a. Sublist

$$1 + 1 = 2$$

$$2 = 2$$

Item

Second paragraph

$$1 + 1 = 2$$

$$2 = 2$$

- b.
 - $1 + 1 = 2$
 - $2 = 2$
 - Sublist Item
 - Second
 - $1 + 1 = 2$
 - $2 = 2$
 - paragraph
- c. Sublist Item
 - $1 + 1 = 2$
 - $2 = 2$
 - $1 + 1 = 2$
 - $2 = 2$
 - Second paragraph

And

- $1 + 1 = 2$
- $2 = 2$

after.

- 3. Structured item
 - $1 + 1 = 2$
 - $2 = 2$
 - $1 + 1 = 2$
 - $2 = 2$
 - Second paragraph

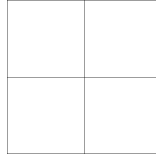
And after.

Checkpoint 10.3 (Checking Proper Indentation In Lists with Images and Tables).

- 1. Structured item

- 2. Structured item

3. Structured item



Second paragraph

4. Structured item

5. Structured item

a b
c d

1 two $[\pi]$
I + I + I + I 5 six
 $2^3 - 1$ VIII

6. Structured item

7. Structured item

a b
c d

Second paragraph

8. Structured item

Checkpoint 10.4 (Deep-nested lists). Ordered list.

1. Level 1, first.
2. Level 1, second.
 - a. Level 2, first.
 - b. Level 2, second.
 - i. Level 3, first.
 - ii. Level 3, second.
 - A. Level 4, first.
 - B. Level 4, second.
 - C. Level 4, third.
 - iii. Level 3, third.
 - c. Level 2, third.
3. Level 1, third.

Unordered list.

- Level 1, first.
- Level 1, second.
 - Level 2, first.
 - Level 2, second.
 - Level 3, first.
 - Level 3, second.
 - Level 4, first.
 - Level 4, second.
 - Level 4, third.
 - Level 3, third.
 - Level 2, third.
- Level 1, third.

11 Layout Configuration Testing

This section is to provide a single page demonstrating all of the combinations that may factor into a problem's layout.

11.1 Inline Exercises

Some "inline" exercises, as distinguished from the "divisional" exercises below.

Checkpoint 11.1. $1 + 1 = 2$

Checkpoint 11.2 (Has a Title). $1 + 1 = 2$

Checkpoint 11.3. Has an introduction.

$$1 + 1 = 2$$

Checkpoint 11.4 (Has a Title). Has an introduction.

$$1 + 1 = 2$$

Checkpoint 11.5. $1 + 1 = 2$

Has a conclusion.

Checkpoint 11.6 (Has a Title). $1 + 1 = 2$

Has a conclusion.

Checkpoint 11.7. Has an introduction.

$$1 + 1 = 2$$

Has a conclusion.

Checkpoint 11.8 (Has a Title). Has an introduction.

$$1 + 1 = 2$$

Has a conclusion.

Divisional Exercises

1. $1 + 1 = 2$

2. (Has a Title) $1 + 1 = 2$

3. Has an introduction.

$$1 + 1 = 2$$

4. (Has a Title) Has an introduction.

$$1 + 1 = 2$$

5. $1 + 1 = 2$

Has a conclusion.

6. (Has a Title) $1 + 1 = 2$

Has a conclusion.

7. Has an introduction.

$$1 + 1 = 2$$

Has a conclusion.

8. (Has a Title) Has an introduction.

$$1 + 1 = 2$$

Has a conclusion.

These are inside an exercisegroup.

9. $1 + 1 = 2$

10. $1 + 1 = 2$

11. Has an introduction.

$$1 + 1 = 2$$

12. Has an introduction.

$$1 + 1 = 2$$

13. $1 + 1 = 2$

Has a conclusion.

14. $1 + 1 = 2$

Has a conclusion.

15. Has an introduction.

$$1 + 1 = 2$$

Has a conclusion.

16. Has an introduction.

$$1 + 1 = 2$$

Has a conclusion.